## PLANAR JANSEN MECHANISM

Figure 1 shows a planar Jansen mechanism. This mechanism is a one degree of freedom, planar mechanism, designed by Theo Jansen, that converts rotational movement of a crank into a smooth stepping motion, and mimics the path of a human's foot when stepping in a forward direction. Furthermore, it allows a wide variety of gait cycles, depending on the configuration of the links.

The mechanism is composed of eleven links, connected by revolute joints, in which the crank $\left(L_{1}\right)$ rotates around point $P_{1}$. Points $P_{1}$ and $P_{2}$ are connected to the ground through two pinned supports. Their global position, in meters, is [0, 0] and [0.380, -0.078], respectively. Each link has a linear mass density of $1 \mathrm{~kg} . \mathrm{m}^{-1}$. Table 1 presents the length of each link, the ratio with respect to the length of the crank, and the respective mechanical properties. The revolute joints and the pinned supports are assumed to be geometrical ideal and frictionless.


Figure 1: Planar Jansen mechanism discretized with Fully Cartesian Coordinates [1].

Table 1: Link lengths, ratio with respect to the crank length, and respective mechanical properties.

|  | Length $[\mathrm{m}]$ | Ratio | Mass $[\mathrm{kg}]$ | Moment of Inertia $\left[\mathrm{kg} \cdot \mathrm{m}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| L1 | 0.150 | 1 | 0.150 | $2.812 \mathrm{E}^{-4}$ |
| L2 | 0.500 | 3.333 | 0.500 | $1.041 \mathrm{E}^{-2}$ |
| L3 | 0.415 | 2.767 | 0.415 | $5.956 \mathrm{E}^{-3}$ |
| L4 | 0.393 | 2.620 | 0.393 | $5.058 \mathrm{E}^{-3}$ |
| L5 | 0.619 | 4.127 | 0.619 | $1.976 \mathrm{E}^{-2}$ |
| L6 | 0.558 | 3.720 | 0.558 | $1.447 \mathrm{E}^{-2}$ |
| L7 | 0.401 | 2.673 | 0.401 | $5.373 \mathrm{E}^{-3}$ |
| L8 | 0.394 | 2.627 | 0.394 | $5.096 \mathrm{E}^{-3}$ |
| L9 | 0.367 | 2.447 | 0.367 | $4.119 \mathrm{E}^{-3}$ |
| L10 | 0.657 | 4.380 | 0.657 | $2.363 \mathrm{E}^{-2}$ |
| L11 | 0.491 | 3.273 | 0.491 | $9.864 \mathrm{E}^{-3}$ |

The system moves under gravity effect ( $-9.81 \mathrm{~m} \cdot \mathrm{~s}^{-2}$ along the global $y$ axis) and starts when the crank $\left(L_{1}\right)$ is at a horizontal position, with its center of mass located at coordinates [0.075, 0]. The initial velocity of its center of mass, in meters per second, is [ $0,-0.47124$ ], which corresponds to a crank angular velocity of $2 \pi$ rad. $\mathrm{s}^{-1}$. The total simulation time is 10 s .

Table 2 Initial Conditions of the Jansen mechanism

| Body | $\boldsymbol{x}$ <br> $[\mathrm{m}]$ | $\boldsymbol{y}$ <br> $[\mathrm{m}]$ | $\boldsymbol{u}_{\boldsymbol{x}}$ <br> $[\mathrm{m}]$ | $\boldsymbol{u}_{\boldsymbol{y}}$ <br> $[\mathrm{m}]$ | $\dot{\boldsymbol{x}}$ <br> $\left[\mathrm{m} \cdot \mathrm{s}^{-1}\right]$ | $\dot{\boldsymbol{y}}$ <br> $\left[\mathrm{m} . \mathrm{s}^{-1}\right]$ | $\dot{\boldsymbol{u}}_{\boldsymbol{x}}$ <br> $\left[\mathrm{m} \cdot \mathrm{s}^{-1}\right]$ | $\boldsymbol{u}_{\boldsymbol{y}}$ <br> $\left[\mathrm{m} . \mathrm{s}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.07500 | 0.0 | 1.0 | 0.0 | 0.0 | -0.47124 | 0.0 | -6.28319 |
| 2 | 0.34967 | 0.15044 | 0.79868 | 0.60176 | -0.53532 | -0.23198 | -2.14128 | 2.84200 |
| 3 | 0.46467 | 0.11144 | -0.40805 | -0.91296 | -0.53532 | 0.23926 | 2.57985 | -1.15306 |
| 4 | 0.51658 | -0.21928 | -0.69504 | 0.71897 | 1.09904 | 1.06246 | -5.59309 | -5.40692 |
| 5 | 0.40158 | -0.18028 | -0.81284 | 0.58248 | 1.09904 | 0.59122 | -3.55103 | -4.95541 |
| 6 | 0.65266 | 0.04171 | 0.37031 | -0.92891 | -0.33829 | 0.77047 | 2.62492 | 1.04642 |
| 7 | 0.56799 | -0.14773 | 0.93758 | -0.34776 | 0.19703 | 0.53121 | 0.98271 | 2.64943 |
| 8 | 0.86179 | -0.38362 | 0.53713 | -0.84350 | 1.33817 | 1.66362 | 4.79241 | 3.05177 |
| 9 | 0.81038 | -0.45517 | -0.85681 | 0.51563 | 2.24018 | 2.19487 | -0.22940 | -0.3812 |
| 10 | 0.65271 | -0.64335 | -0.95858 | -0.28481 | 2.32390 | 2.12472 | 0.12671 | -0.42648 |
| 11 | 0.49548 | -0.54873 | 0.64224 | 0.76650 | 2.28180 | 2.05477 | -0.34102 | 0.28573 |

Figure 2 shows the $x$ and $y$ coordinates of point $P_{3}$ obtained during the computational simulation performed using the fully Cartesian coordinates formulation [1] with custom scripts written in Python 3.7. The technical details of the simulation, integrator type, hardware used, and software specifications are presented in Table 3.


Figure 2 Planar Jansen mechanism: $x$ and $y$ coordinates of point $\mathrm{P}_{3}$.
Figure 3 shows the variation of the mechanical energy of the system and the position and velocity constraints violations registered during the simulation. The constraints violations were calculated as the norm of the vector of kinematic constraints at the corresponding level (position or velocities). The mechanical energy is obtained as the sum of the kinetic and potential energies of the mechanism.


Figure 3: Variation of the mechanical energy of the system and the constraints violations of position and velocity (logarithmic scale).

The goal of this benchmark problem is to simulate the dynamic behavior of a planar mechanism composed of 11 rigid bodies, several closed loops and only one degree of freedom. A reference text file with the results is available, for comparison purposes. The file is composed of six columns. The first one represents the simulation timestamp, from 0 s to 10 s . The second and third columns contain the $x$ and $y$ coordinates of point $P_{3}$ during the motion. The fourth one is the mechanical energy of the system, and the two last ones represent the norm of the constraints violations of position and velocity.

Table 3 Technical details for the problem, integrator, hardware, and software.

| Attribute | Value |
| :---: | :---: |
| Integrator | Adams / BDF |
| Solver type | Variable time-step |
| Accuracy | $3 \times 10^{-6}$ |
| Relative tolerance | $10^{-10}$ |
| Absolute tolerance | $10^{-10}$ |
| CPU time | 76.17 s |
| CPU/GPU | Intel(R) Core ${ }^{\mathrm{TM}} \mathrm{i7}-8700 \mathrm{CPU} 3.20 \mathrm{GHz}$ |
| Operating system | Windows10 |
| Formulation | Fully Cartesian coordinates |
| Equations of Motion | DAEs index-3 |
| Programming language | Python |

## References

[1] I. Roupa, S.B. Gonçalves, M.T. Silva, Dynamic Analysis of Planar Multibody Systems with Fully Cartesian Coordinates, Proc. Int. Conf. Multibody Syst. Dyn. (2018) 15.

